

Classical and quantum cross-section for black hole production in particle collisions

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Abstract

We suggest a simple model to study the problem of the black hole production in particle collisions. The cross-section for the classical and quantum production is analysed within this model. In particular, the possibility to form a black hole in collision of low energy particles (or at large impact parameter) via the quantum tunneling mechanism is pointed out. It is found that, in this model, the geometric cross-section gives a good estimate for the production at low and high energies. We also reconsider the arguments in favor of exponential suppression for the production of trans-Planckian black hole and conclude that no such suppression in fact appears. Analyzing the probability for the black hole production we point out on the importance of the back-reaction and reevaluate the contribution of the black hole formed in gravitational collapse to the Euclidean path integral.

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1 Introduction

For long time black holes have been an interesting object for theoretical exercises that lies on the intersection of General Relativity and Quantum Mechanics. Many unusual features of this object have been revealed in the purely theoretical study in the last three decades. Most significantly, this includes the Hawking radiation and the thermodynamical description of the black hole [1].

In spite of the considerable progress the fundamental theory of black holes never had a chance to become a field of science having a touch with experiment. Nevertheless, the numerous attempts to analyse the possibility to create a black hole in laboratory have been made in the past [2]-[8]. It became clear that the problem of producing a black hole in, say, a particle collider is not just a problem of reaching high energy. The analysis of collisions at trans-Planckian energy in the so-called eikonal regime reveals no black hole production. In this regime the particles fall to each other at large transverse separation and do not produce strong gravitational field in the transverse direction. The problem thus is in overlapping the wave functions of the colliding particles close enough to turn on the strong gravitational interaction.

On the other hand, the total effect is usually expected to be visible only when the energy of the particles becomes Planckian or even trans-Planckian. Since the Planckian scale lies far beyond the Standard Model scale, the problem of producing black holes in particle collisions was always considered of a purely academic interest. This situation has a chance to change. There were suggested phenomenologically viable models [9],[10] of multi-dimensional gravity in which the Planck scale can be as low as in a TeV range. This opens an exciting (yet requiring further examination) possibility to enter the Planckian region in realistic future colliders and makes the black hole production one of the most urgent theoretical problems. That is why the issue of producing black holes in LHC or by cosmic rays becomes increasingly popular [11]-[24].

As for the theoretical aspects of this problem they are surprisingly undeveloped. Estimating the cross-section for the production of black hole in collision with total energy E one typically takes it to be given by the horizon area [25], $\sigma_{\text{prod}} \sim \pi r_g^2$, where $r_g = 2GE$ is the radius of the created black hole. This is the so-called geometric cross-section. The derivation of this estimate is based mainly on intuition and was not actually justified theoretically. It is also a part of general believe that the gravitational collapse at trans-Planckian energies does not much differ from the collapse as we know it in General Relativity. Indeed, the expected radius $r_g \sim GE$ of the produced black hole is much bigger than the Planck length. Therefore, the Quantum Gravity corrections can be considered as small and not playing the key role in this process.

This, however, does not guarantee that the naive picture is automatically correct. The validity of the geometric cross-section was criticized in [26] (see also [27]) as not actually providing us with the correct value for the semiclassical cross-section. It was argued in [26] that semiclassical treatment of the problem reveals the exponential suppression by the factor $\exp(-4\pi GE^2)$. If correct, such suppression would make negligible the production of trans-Planckian black holes. This conclusion contradicts the intuition based on the

classical collapse picture. It is therefore principally important to better understand this issue.

In this paper we partially fill the gap in the theoretical study and give a classical and quantum analysis of the problem of black hole production in particle collisions in a simple (still meaningful) model. An advantage of this model is that it suggests a well-defined procedure for actually computing the classical and quantum cross-section for the black hole production. In particular, we obtain a justification for the geometric cross-section in a wide range of the energy. For lower energy particles our model suggests an interesting possibility to produce black hole in a purely quantum process when particles tunnel through the effective potential barrier separating them in the radial direction. Since the wavelength of such particles is much bigger than the critical impact parameter a quantum treatment (like the one we perform) is necessary. The cross-section for this process is given by the geometric cross-section.

We also analyse in detail the arguments of paper [26] in favor of the exponential suppression. Two approaches were suggested in [26]: using the path integral and the statistical approach. We reconsider both approaches and find no such suppression to actually appear. Our counter-arguments are quite instructive and include the important issues of backreaction of the Hawking radiation as well as correct evaluation of the contribution of the black hole formed in gravitational collapse to the Euclidean path integral.

2 A model picture for the black hole production

The difficulties of the analysis of the black hole formation in the process of colliding two particles lies in the fact that the classical two-body problem is not solved in General Relativity (the only exception is $(2 + 1)$ -dimensional case [28], [29] ; it may serve to guide our intuition in higher-dimensional case but should be taken with some caution: gravity in 3 dimensions does not propagate and thus describes a very special type of interaction). In order to overcome this difficulty we suggest a simple model in which the black hole production (or, actually, a process equivalent to it) can be studied in a rather straightforward way.

Let us start with a classical picture of two particles with energy E_1 and E_2 respectively moving towards each other at certain impact parameter b . For simplicity we will be considering ultra-relativistic particles ($m_i/E_i \ll 1$) with velocity close to the speed of light. Also, we assume that the motion of the system as whole happens in one plane. In Newtonian mechanics this system would be equivalently described in the center mass coordinate system as a test particle with energy $\omega = \frac{E_1 E_2}{E_1 + E_2}$ falling on the gravitating center of mass $M = E_1 + E_2$. In General Relativity this picture is more complex due to the non-linear nature of the gravitational interaction. In order to include the gravitational interaction in the game and put everything into a still analytically tractable scheme we replace the above picture with an approximate one. We consider a gravitating center with mass M which creates around it the gravitational field described by Schwarzschild metric with gravitational radius $r_g = 2GM$, and a test particle with energy ω falling

into the center from infinity at the impact parameter b . It should be noted that in this picture the black hole horizon at $r = r_g$ is absolutely fictitious. However, we say that the actual horizon forms when the test particle crosses $r = r_g$. The latter thus signals for the black hole production in the original picture. This model is of course an approximation. Its main advantage though is that it suggests a well defined procedure to compute, both classically and quantum mechanically, the cross-section for the black hole production when two particles collide. Also, the formulas relevant to this picture are already available in the literature studying the classical and quantum particle scattering by black hole, a review of the existing literature can be found in [30]. We just have to give them a new interpretation. However, this model is not a substitution for the desirable analysis of non-linear process of actual gravitational collapse by the colliding particles. The earlier works in this direction include [31], [32], [33], [34]. An interesting recent work is [35]. Our model should be considered as complimentary to such analysis.

The classical radial motion of the test particle is determined by the geodesic equation (we put $c = 1$)

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{r_g}{r}\right)^2 b^2 \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_g}{r}\right)\right) , \quad (2.1)$$

where we used the fact that for a ultra-relativistic particle, its energy ω and angle momentum L (computed with respect to the gravitating center) are related as $L/\omega = b$. For massive particle there is an extra term in the geodesic equation (2.1). However, in the ultra-relativistic ($m/\omega \ll 1$) limit this term can be neglected if the impact parameter satisfies condition $b \gg Gm$ thus excluding values of the angle momentum close to zero. Note that for the present situation the black hole mass $M \sim \omega$ and hence $(Gm)/r_g \ll 1$. So that it does not put a serious restriction to our model. The geodesic trajectories described by (2.1) are well studied. For a particle coming from infinity the crucial relation is the relation between $1/b^2$ and the maximal value of the effective potential $V(r) = \frac{1}{r^2} \left(1 - \frac{r_g}{r}\right)$. This potential takes its maximal value at $r_m = \frac{3}{2}r_g$ and $V(r_m) = \frac{4}{27r_g^2}$. Therefore, for impact parameter $b < b_{\text{cr}} = 3\sqrt{3}/2r_g$ the test particle is captured by the gravitating center and eventually falls into horizon. Having in mind the original picture of the colliding particles we say that there forms a black hole with horizon radius r_g . In the case $b > b_{\text{cr}}$ the particle just scatters off the center and the gravitational capture does not happen. In the original picture, this would correspond to particles passing each other without actually forming the black hole. Thus, classically, the cross-section for the black hole formation is given by

$$\sigma_{\text{cl}} = \pi b_{\text{cr}}^2 = \frac{27}{4} \pi r_g^2 = 27 \pi G^2 M^2 . \quad (2.2)$$

Thus, this model predicts that, classically, two colliding particles form a black hole if they pass each other at the shortest distance $b < b_{\text{cr}} = 3\sqrt{3}/2r_g$ (where r_g is the gravitational radius for the system of these two particles). Note, that this is a little bigger than one could expect from, say, Thorne's hoop conjecture. This is because the particles need to get over the potential barrier separating them and staying just outside the effective horizon.

Then, having reached the other side of the barrier they fall to each other without any other obstacles.

This picture can be also analysed quantum mechanically. The relevant processes, black hole scattering and absorption, were well studied in the past and here we give a brief summary with the re-interpretation according to our picture of the black hole production. Decomposing quantum field in spherical harmonics, $\Phi_{lm} \sim \frac{u_l(r, \omega)}{r} e^{-i\omega t} Y_{lm}(\theta, \phi)$, one arrives at the radial wave equation

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - U_l(r) \right) u_l(r, \omega) = 0, \quad (2.3)$$

which is a quantum mechanical analog of the classical equation (2.1). We denote $\frac{d}{dr_*} = (1 - \frac{r_g}{r}) \frac{d}{dr}$ and

$$U_l(r) = (1 - \frac{r_g}{r}) \left(\frac{l(l+1)}{r^2} + \frac{r_g(1-s^2)}{r^3} \right), \quad (2.4)$$

where s is spin of the particle (below we put $s = 0$), is the effective potential similar to the potential in eq.(2.1). The equation (2.3) describes waves scattering by the potential (2.4). The relevant modes are defined to be in- and out-going at infinity $r_* \rightarrow \infty$, $u_l(r, \omega) \sim A_{\text{out}}(\omega) e^{i\omega r_*} + A_{\text{in}}(\omega) e^{-i\omega r_*}$, and only out-going at horizon $u_l(r, \omega) \sim e^{-i\omega r_*}$, $r_* \rightarrow -\infty$. Again, in this picture we say that the black hole production takes place if the wave is absorbed by the effective horizon around the gravitating center of mass M .

The probability of wave to penetrate through the potential barrier is

$$\Gamma_{l, \omega} = 1 - \left| \frac{A_{\text{out}}}{A_{\text{in}}} \right|^2.$$

Note, that it is probability for a spherical wave. Since the actual wave at infinity looks more like a plane wave the latter should be decomposed on the spherical modes

$$e^{-i\omega z} = \sum_{l=0}^{\infty} K_l(\omega) Y_{l0}(\theta, \phi), \quad K_l(\omega) = \frac{i^l}{2\omega} [4\pi(2l+1)]^{1/2}. \quad (2.5)$$

The cross-section to capture the test particle is then defined as follows

$$\sigma_{\text{quant}} = \sum_l |K_l|^2 \Gamma_{l, \omega}. \quad (2.6)$$

It is also convenient to consider each partial wave characterized by angular momentum l as falling on the gravitating center at impact parameter b ,

$$b = (l + \frac{1}{2}) \frac{1}{\omega}.$$

In the high energy limit the geometrical optics analysis based on equation (2.1) becomes a good approximation. In this limit the cross-section (2.5) approaches the classical value (2.2). One can also compute $1/\omega^2$ -corrections. The result then reads

$$\sigma_{\text{quant}} \simeq \frac{27}{4}\pi r_g^2 - \frac{2}{3}\pi \frac{1}{\omega^2} . \quad (2.7)$$

The horizon absorbs partial waves with impact parameter $b \leq \frac{3}{2}r_g$ in this regime.

It is also interesting to analyse the opposite limit of small ω . As argued in [36] it is actually the limit of small $\omega r_g/l$, i.e. $b \gg r_g$ in this case. Remarkably, the cross section (2.6) approaches a finite number in this regime equal to the horizon area²

$$\sigma_{\text{quant}} \simeq \pi r_g^2 . \quad (2.8)$$

Naively, one would expect this to vanish. The transmission probability $\Gamma_{l,\omega}$ does vanish for small ω as $\sim \omega^{2l+2}$ and dominates for s-wave. However, the contribution of the s-wave in the in-falling plane wave diverges $|K_l|^2 \sim \frac{1}{\omega^2}$, as is seen from (2.5). The two tendencies compensate each other in (2.6) for the s-wave. This results in the finite cross section (2.8). Since only the wave that approaches the horizon radially is absorbed it is not surprising that the cross-section becomes equal to the horizon area. We should also note that the penetration through the barrier of the wave with small ω (or large impact parameter, $b \gg r_g$) is classically forbidden! Therefore, the cross section (2.8) is entirely due to the quantum tunneling effect.

We see that the geometric cross-section gives a rather good estimate for the black hole production in a wide range of energy ω . The ratio of the actual cross-section and the geometric cross-section appears to be a slow changing function of the energy.

Does this simplified picture give a correct description of what happens in the actual gravitational collapse when two particles collide? We do expect this picture to be a good approximation when one of the particles has much bigger energy than another. In the case of the approximately equal energy this picture is still valid when two particles collide at the impact parameter of the same order as the effective gravitational radius. The gravitational field is strong and essential when particles come close to each other at distance $r \sim r_g$ and the total gravitational field indeed can be approximated by the field created by total mass located somewhere in between. This is also the case for the high-energy collision. In fact, in this regime the geometrical optics analysis becomes a good approximation. Therefore, the validity of our model then reduces to the same question for the classical problem of two colliding bodies in General Relativity. In case of colliding black holes this issue was studied in the literature (see for instance [41] and references therein) and good agreement with exact (numerical) solution was found.

The validity of the picture is less obvious when the impact parameter is relatively large compared to the gravitational radius r_g . Still, the effective potential we considered models the actual potential between the particles. Classically, the collapse is forbidden

²This is a quite old result first obtained by Starobinsky [37], see also a nice analysis given in a paper by Unruh [38]. The higher-dimensional analysis is given recently in [39].

in this case. However, we think that our analysis opens an important possibility to produce a black hole by colliding low energy particles via the quantum tunneling. This regime is closer to the real situation when the wavelength of the particles is much bigger than the critical impact parameter thus justifying the quantum mechanical treatment we performed.

There are few directions for further improving our calculation. First of all, the black hole formed in collision of particles is expected to be rotating. Therefore, the rotation should be included in our picture. The corresponding classical and quantum analysis is given in the existing literature, see for example [40] and [30], the extension of our consideration to this case is quite straightforward.

The Quantum Gravity corrections neglected in the above consideration can be taken into account. These corrections result in shifting the location of the Schwarzschild horizon, so that the new location r_q is given by (an example of the quantum corrected Schwarzschild metric is given in [42])

$$r_q^2 = (2GM)^2 + \# l_{pl}^2 \quad , \quad (2.9)$$

where $l_{pl} \sim \sqrt{G}$ is Planck length and $\#$ is some number which may also include the slow changing with energy logarithmic term $\ln M$. The geometric area is now given by πr_q^2 . If the total energy M is much bigger than $M_{p1} \sim G^{-\frac{1}{2}}$ the quantum shift can be neglected. However when $M \simeq \text{few } M_{p1}$ (it is expected that the minimum mass to produce black holes on LHC is $M_{\min} \simeq 5M_{p1}$, see [14]) both terms in (2.9) become equally important. This may change the numerical value of the cross-section (see also discussion in [20]).

Another important issue missed so far is the gravitational radiation. Indeed, the test particle falling onto the black hole is expected to radiate some amount of its energy via producing the gravitational waves (gravitons). The standard estimate (see [41]) for this amount is

$$E_{\text{rad}} \sim 0.01 \omega^2 / M \quad . \quad (2.10)$$

The same estimate is valid for the gravitational radiation in collision of two black holes [41]. The main part of this energy goes to the quasi-normal oscillations. The latter are excited as the particle passes through the peak of the potential barrier. For high energy this peak is at $r = \frac{3}{2}r_g$. The actual mass of the black hole formed by colliding the particles can be a certain fraction of the value $M = E_1 + E_2$. Provided the same estimate (2.10) is applicable to the radiation emitted in collision of particles the fraction of the total energy lost via radiation is quite small, $E_{\text{rad}} \sim 10^{-3}M$. So that there could be a small correction to our formulas due to this energy loss.

Interestingly, in the case of low energy collision when the particles tunnel through the barrier separating them they do not actually pass through the peak of the potential barrier and hence lose less energy. Therefore, in some cases it might be more efficient to produce black holes by lower energy particles via the quantum tunneling rather than in the high energy collapse.

3 Is the black hole production exponentially suppressed?

The applicability of the geometric cross-section to the process of the black hole production in collision of particles with energy exceeding the Planck energy was criticized in [26], [27]. It was argued in these papers that the semiclassical analysis gives rise to the exponential suppression of the cross-section as

$$\sigma_{\text{semicl}} \sim e^{-4\pi GM^2} \quad (3.1)$$

that makes negligible the production of black hole with large mass $M \gg M_{\text{pl}}$. Note that this conclusion is quite counter-intuitive. Indeed, our experience based on the study of classical equations of General Relativity says that the concentration of large energy in small enough space-time region inevitably leads to formation of a black hole. Also, the fact that the ratio of M/M_{pl} is large means that the expected size r_g of horizon is much bigger than the Planck length $l_{\text{pl}} \sim \sqrt{G}$ and hence the Quantum Gravity corrections to the classical process must be small not producing the exponential factors as in (3.1). The latter argument also means that the semiclassical analysis should be a reliable approximation to describe this process. Note, that for the model considered in previous section the validity of the semiclassical approximation for high energy justifies the fact that the classical geometric optics analysis (2.1), (2.2) gives the right answer in this case. No exponential factor like (3.1) arises there.

In this section we re-consider the arguments of paper [26] and show that the consistent treatment of the problem does not lead to the appearance of exponential factors similar to (3.1). Two approaches were suggested in [26].

3.1 Statistical approach

The black hole of mass M can be viewed as a macroscopic object realized by a large number of micro-states (H), $\mathcal{N} = \exp(S_H)$, determined by the entropy $S_H = 4\pi GM^2$. The probability to create the black hole in collision of few particles can be obtained by summing up the probabilities to create a black hole at a given micro-state H . Thus, the total probability is proportional to \mathcal{N} . On the other hand, for a given H each such probability, by the CPT symmetry, is related to the probability of the reverse process of the black hole decay into few (anti)particles. The latter can be estimated by the Gibbs formula provided the black hole decays thermally with temperature $T_H = 1/(8\pi GM)$. These reasonings led in [26] to derive the total probability in the form

$$\begin{aligned} P(\text{few} \rightarrow \text{black hole}) &\sim \mathcal{N} P(\text{black hole} \rightarrow \text{few}) \\ &\sim \exp\left(S_H - \sum_i \frac{E_i}{T_H}\right) \quad , \end{aligned} \quad (3.2)$$

where E_i are the energies of individual particles the black hole decays to. The first term in (3.2) is due to the black hole degeneracy while the second term is the probability of the

reverse (decay) process³. The black hole decays until it disappears, therefore $\sum_i E_i = M$. Then, what stands under the exponent in (3.2) is $(S_H - T_H^{-1}M)$ which is equal to $(-S_H)$ for Schwarzschild black hole. This is what was obtained in [26].

What is missing in the above consideration is the fact that a radiated particle causes certain effect of the back-reaction on the black hole. Namely, mass of the black hole and its temperature change respectively. This effect is not difficult to take into account. By the moment the black hole of initial mass M has radiated particles with total energy ω its mass becomes $(M - \omega)$ and the inverse Hawking temperature is $T^{-1}(\omega) = 8\pi G(M - \omega)$. Therefore, the probability to radiate next particle with small energy $d\omega$ is

$$P(\omega, d\omega) \sim \exp(-T^{-1}(\omega)d\omega) \quad .$$

The total decay probability then is the product of these probabilities for all individual particles until the black hole disappears completely. The equation (3.2) thus should be replaced by the following equation

$$P(\text{few} \rightarrow \text{black hole}) \sim \exp(S(M) - \int_0^M 8\pi G(M - \omega)d\omega) \quad . \quad (3.3)$$

It is easy to check that the expression under the exponent vanishes identically for the Schwarzschild black hole.

One might argue [46] that in our modification of the calculation given in [26] the decay of black hole is a slow step-wise decay while for the process of the formation of black hole in few-particle collision it is more relevant to consider a decay into few (e.g. two) particles at once. However, it is clear that the latter type of decay can be hardly called thermal and hence it is not eligible at all to use (as it was done in (3.2)) the thermal form for the probabilities. Moreover, after a minor modification our calculation can be applied to a decay on arbitrary large pieces not assuming at all the thermal character of the decay. Indeed, on general grounds, for a highly degenerate system the probability to radiate a particle is proportional to the exponent of the corresponding change of the entropy⁴. Applying this to black hole, we find that after the i -th particle has been radiated the entropy of black hole changes on $-\delta S_i$ so that the discussed probability is

$$P(\text{few} \rightarrow \text{black hole}) \sim \exp(S_H - \sum_i \delta S_i) \quad , \quad (3.4)$$

where number of radiated particles can be arbitrary (two, three or a hundred). Since the black hole is supposed to radiate completely in the reverse process, we have that $\sum_i \delta S_i = S_H$. This again means no exponential suppression.

We should note that it was actually expected in [26] that the back-reaction may play certain role in the discussed semiclassical calculation and lead to some modification of the

³In [14] it was argued that the time reversal process should involve a white hole rather than a black hole. We leave aside this possibility assuming that what should stay in (3.2) is indeed the decay probability of the object formed in the direct process, i.e. of the black hole. See also discussion in [27].

⁴Note, that a similar idea has been used in [47] to describe the Hawking radiation when the back-reaction of the radiated particles is taken into account.

formula for probability. However, this was argued to make a relatively small effect which just changes the possible pre-factor in (3.1). We see however that the back-reaction is in fact crucial for the considered process and removes the exponential factor completely.

3.2 Path integral approach

In this approach, computing the probability (or transition amplitude) for the desirable process (**few particles** \rightarrow **black hole**) it is suggested in [26] that one has to deal with the path integral over metric and matter fields subject to appropriate conditions at $t = -\infty$ (few colliding particles) and at $t = +\infty$ (black hole). Evaluating path integrals one normally shifts the t -integration to the complex half-plane: $t \rightarrow t - i\tau$. This results in considering the Euclidean section of the space-time. In the analysis given in [26] it is assumed that the Euclidean section representing the contribution of the black hole is the eternal black hole instanton and the path integral is given semiclassically by

$$P(\text{few} \rightarrow \text{black hole}) \sim \exp(-I_E[g]) \quad , \quad (3.5)$$

where $I_E[g]$ is the gravitational action evaluated on the instanton. The Euclidean black hole instanton can be viewed as a section of complex space-time of eternal black hole by the plane $t = 0$ passing through the bifurcation point at $r = r_g$. The instanton then is known to be regular manifold with abelian isometry generated by vector ∂_τ . This isometry has a stationary point at $r = r_g$. The regularity at this point requires the Euclidean time τ to be periodic with the period being $8\pi GM$ for the Schwarzschild black hole. The Euclidean action then reduces to the boundary term

$$I_E[g] = -\frac{1}{8\pi G} \int_{r=\infty} (K - K_0) \quad (3.6)$$

evaluated over boundary at $r = \text{const} \rightarrow \infty$, K is the extrinsic curvature of the boundary. In order to regularise the gravitational action one normally subtracts the flat space contribution K_0 . Defined in this way action (3.6) is known [43] to be equal to

$$I_E[g] = S_H = 4\pi GM^2 \quad .$$

This again seems to indicate the exponential suppression (3.1) of the semiclassical probability. According to [26] this is due to the exponentially small contribution of the black hole to the total probability.

The above analysis is perfectly suitable to describe the spontaneous creation of a black hole (or, say, creation of black hole pairs in external field, see for example [44], [45]) when the created space-time can be matched to (a part of) the Penrose diagram of eternal black hole including the bifurcation point. The exponential semiclassical suppression then is expected since the process is forbidden classically. This is however not the case for the black hole formed in gravitational collapse⁵. In this case the resultant space-time

⁵Otherwise, it would be applicable to the gravitational collapse of stars making the formation of black hole in this process quantum mechanically impossible.

which we could call the black hole matches only to certain region of the Penrose diagram of the eternal black hole that does not include the bifurcation point. Therefore, the Euclidean instanton of eternal black hole is not appropriate for this situation. Considering a $t = \text{const}$ slice of the total space-time and extending it to a slice of a complex space-time we find that it covers only a part of the Euclidean instanton for $r \geq r_g + \epsilon$, where ϵ is non-zero quantity measuring on $t = \text{const}$ slice the distance between the “surface” of the collapsing system and the would-be horizon. For certain value of t this distance vanishes and the black hole actually forms. In the case under question the “surface” would be formed by trajectories of the colliding particles. The region $0 \leq r \leq r_g + \epsilon$ is inside the collapsing “body”. It can be modeled by space with flat metric. The complex space-time and its Euclidean section have the usual meaning for flat space. The Euclidean instanton relevant to the collapse is thus a part of the eternal black hole instanton and a flat disk glued together at $r = r_g + \epsilon$. The extrinsic curvature has a jump on the surface $r = r_g + \epsilon$. This jump can be thought as arising due to the stress tensor of the collapsing matter. This is the picture arising in the collapse of spherical shell [48]. Evaluating the gravitational action the jump in the extrinsic curvature should be taken into account, so that we arrive at the action

$$I_E[g] = -\frac{1}{8\pi G} \left(\int_{r=r_g+\epsilon} (K - K_0) + \int_{r=\infty} (K - K_0) \right) , \quad (3.7)$$

where all K are defined with respect to the normal vector directed to large r . In order to single out the contribution of the hole itself we take the limit $\epsilon \rightarrow 0$. The integral of K_0 at $r = r_g + \epsilon$ vanishes while that of K is non-zero and equals to the minus entropy of the black hole [49]

$$-\frac{1}{8\pi G} \int_{r=r_g+\epsilon} K = -S_H .$$

Since the contribution of the external boundary is the same as before, $+S_H$, we conclude that the gravitational action (3.7) vanishes identically (a similar calculation in the context of the stretched horizon approach (membrane paradigm) was done in [50]). This removes the dangerous exponent in (3.5) and makes no suppression to the probability. Clearly, this is because the actual contribution of the black hole (formed in the gravitational collapse) to the total probability is of order of one. It is also consistent with the fact that the considered process of the black hole production via gravitational collapse (the collision of particles is an example of such process) is classically allowable (see also [14]).

4 Conclusion

The production of black holes in collision of particles is an interesting and technically difficult problem. Complete solution should involve the classical and quantum analysis of non-linear gravitational interaction between the particles. In this paper we make a step towards this solution and suggest a simple model in which the interaction is modeled by certain effective potential. Within this model we analyse the cross-section for the

production. It is found that the geometric cross-section gives a rather good estimate at low and high energies. At low energy the process of the black hole production is forbidden classically and goes via the quantum mechanical mechanism of the under-barrier tunneling.

In the second part of the paper we resolve the issue of the exponential suppression of the trans-Planckian production of black holes. Our analysis shows that the consistent treatment of the approaches suggested in [26] reveals no exponential suppression. This conclusion is based on the following observations: i) the Gibbons-Hawking calculation used in [26] to estimate the black hole contribution to the Euclidean path integral is *not* appropriate if the black hole was formed in gravitational collapse; ii) calculating the probability of the reverse process of black hole decay the backreaction of the radiated particles on the black hole geometry should be properly included. Provided these issues are properly taken into account there appears no exponentially small terms in the probability to produce black hole at trans-Planckian energy.

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